The Whereabouts of Reliability Education: Challenges & Opportunities

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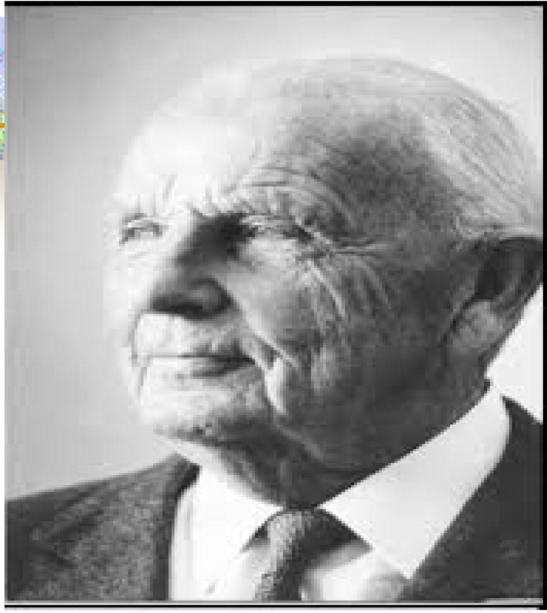
Reliability Education

- Reliability is a discipline that has been studied for several decades.
- Today several dozen graduate programs in the US and hundreds worldwide offer reliability courses, and some universities have entire reliability programs.
- There is a gap between reliability theory and practice, between school and industry, book knowledge and real world applications.
- Due to changes in technology, the expectation for a reliability engineer has been changing and getting higher.

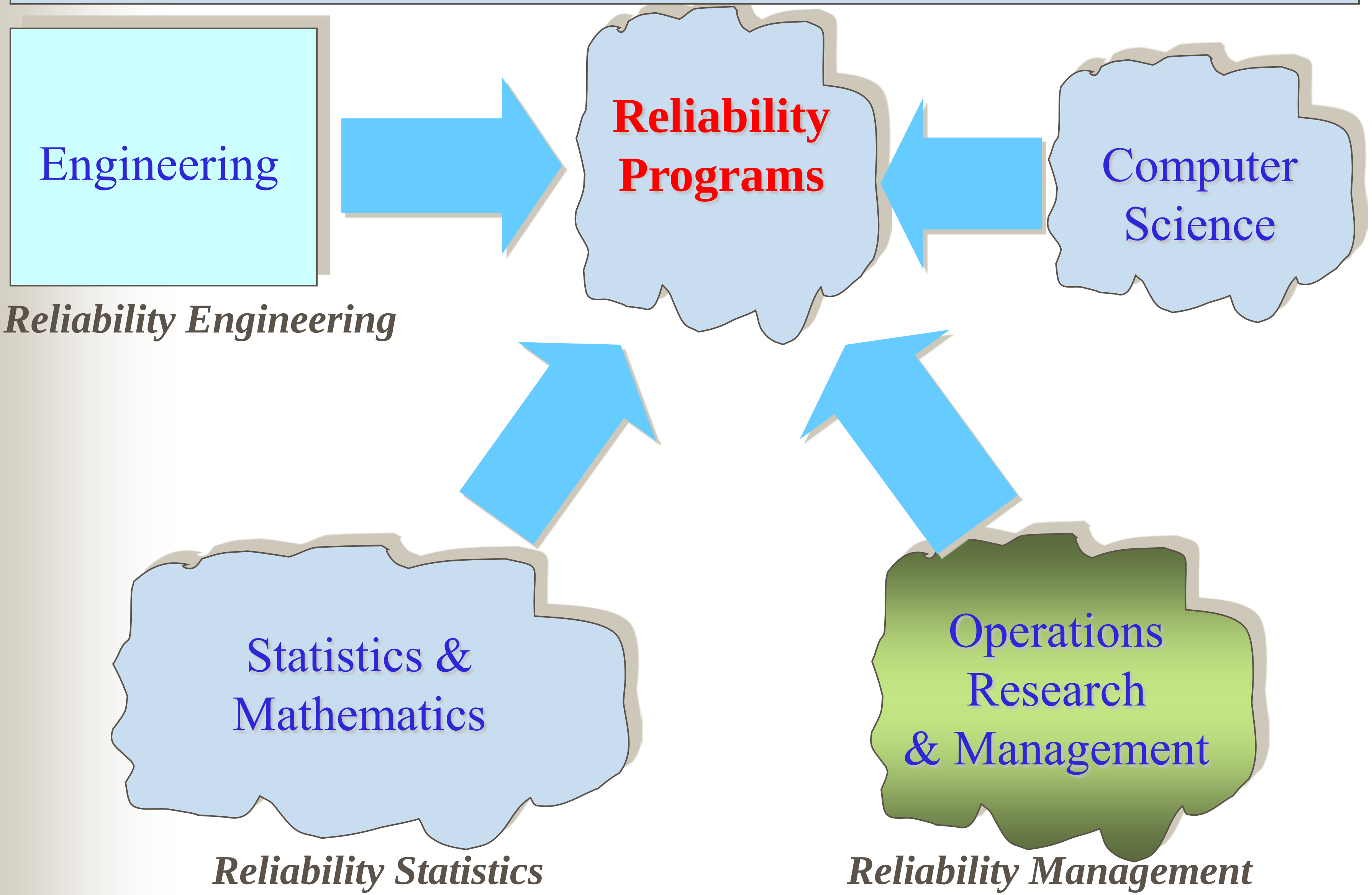
Some Reliability books in … 1960s



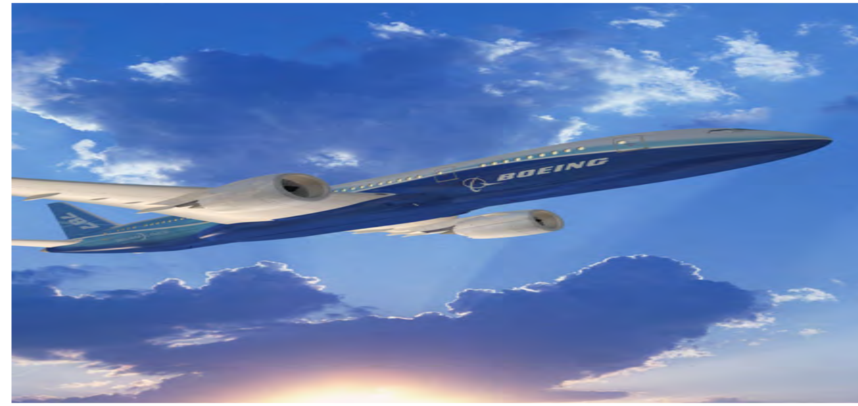
Wallodi Weibull 1887-1979
Photo by Sam C. Saunders

- Igor **Bazovsky** (1961), Reliability Theory and Practice
- D. K. **Lloyd** and M. **Lipov** (1962), Reliability Management, Methods and Mathematics
- N. H. **Roberts** (1964), Mathematical Models in Reliability Engineering
- G. H. **Sandler** (1964), System Reliability Engineering
- R. B. **Barlow** and F. **Proschan** (1965), Mathematical Theory of Reliability

Reliability Programs



In today’s global market, the only way to stay ahead of the competition is to provide:



- Better products!
- Better service!
- Better customer experience every time!

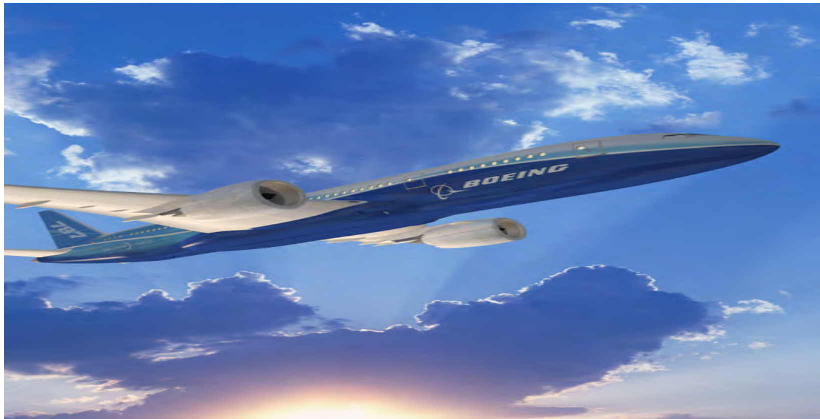
Boeing 787

Sample 3D TV

Reliability Computing

- Reliability requirement: **0.999999999**

- "The airplane systems and associated components …must be designed so that the occurrence of any failure condition which would prevent the continued safe flight and landing…is extremely improbable (1 per billion flights~**10^{-9}**). Compliance… must be shown by analysis…"
    - FAA Federal Aviation Regulations 25.1309



Reliability Challenges From Theory to Practice

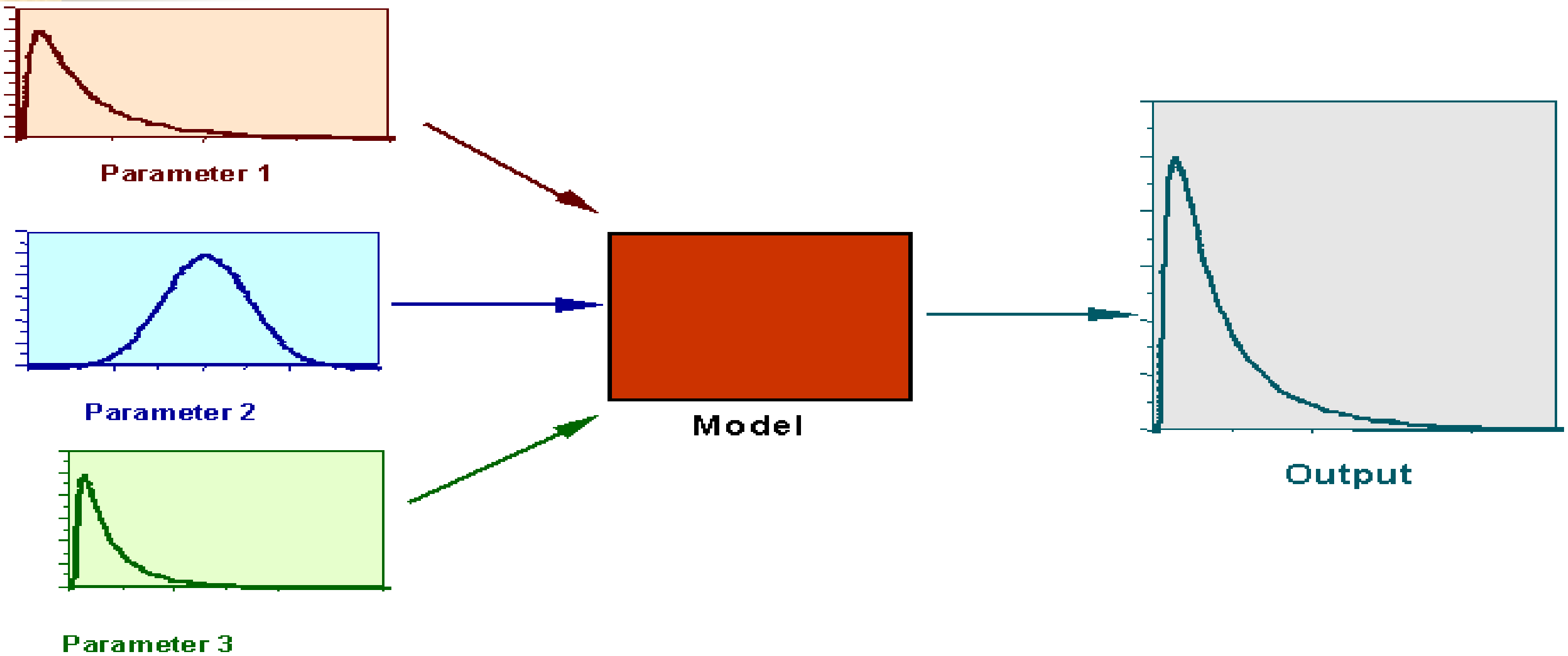
- **DATA QUALITY**

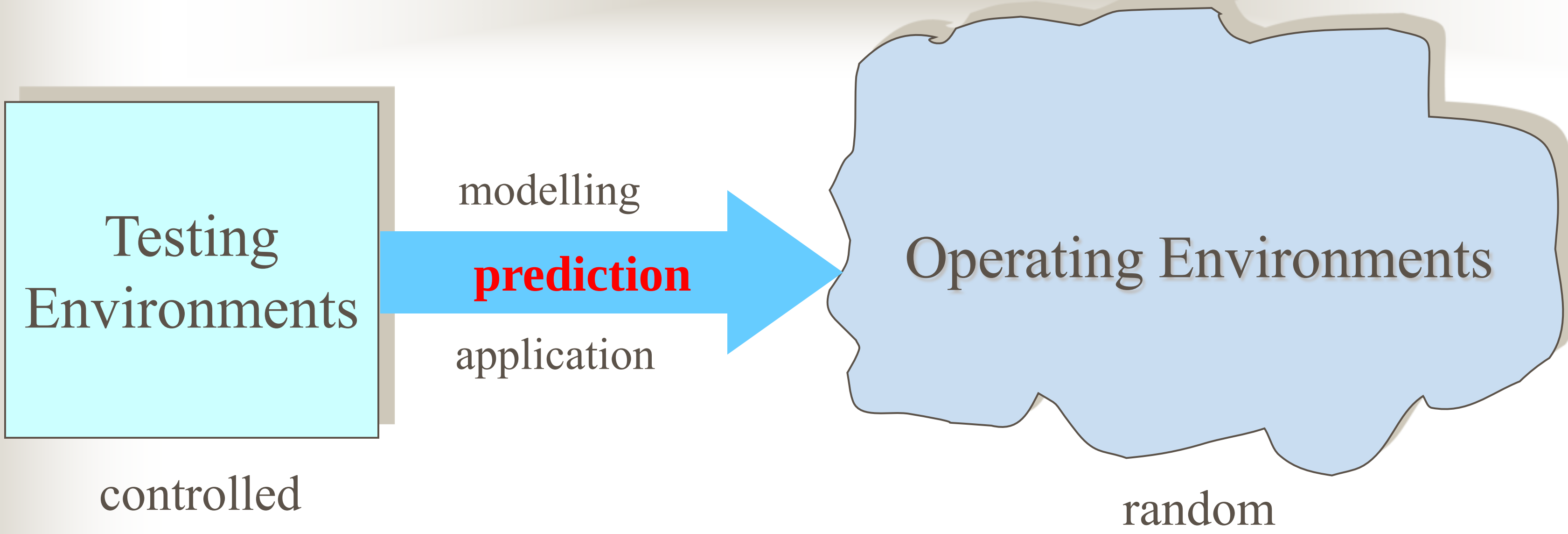
  The Data of Everything!

Key Factors for Data Quality

Reliability Challenges From Theory to Practice

- **PREDICTIVE MODELING**
  - * **The Uncertainty in Modeling!**
  - * **What Models Should Be Used?**

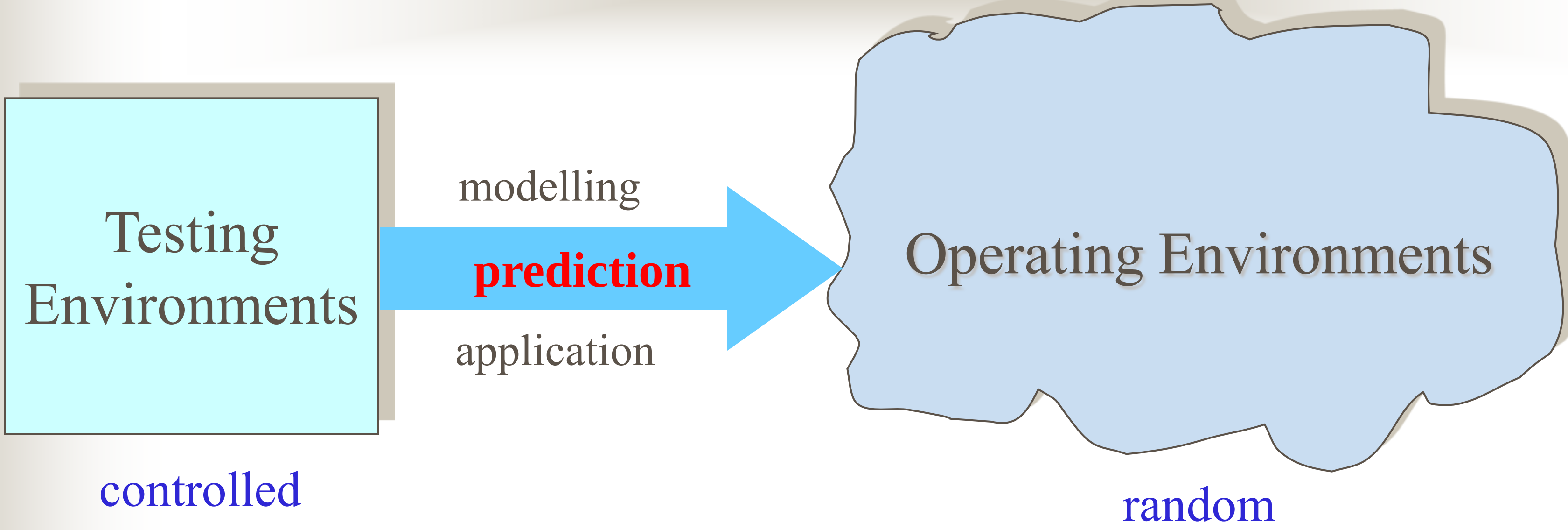




Predictive Modeling

based on data and statistical methods

"Prediction is difficult, especially when it's about the future!"



- Many reliability studies:

Controlled Environment ≈ Operating Environment

- Systemability model

$$\eta = \begin{cases} 1 & \text{Controlled environment} \\ f(\eta) & \text{Operating environment} \end{cases}$$

Reliability -- Definition

- The probability that the system is still operating at time t.

$$R(t) = \int_t^{\infty} f(s)ds = e^{-\int_0^t h(s)ds} = e^{-H(t)}$$

where $f(t)$ probability density function

$h(t)$ failure intensity rate.

Systemability -- Definition

- The probability that the system is still operating at time t subject to the *uncertainty* of the *operating environments*.

  The *systemability* function is [Pham,2005]:

$$R_s(t) = \int_{\eta} e^{-\eta \int_0^t h(s)ds} dF(\eta)$$

where F is a distribution function of η.

- Systemability approximations using Taylor series:

$$R_s(t) = \int_\eta e^{-\eta H(t)} dF(\eta)$$

$$R_s(t) = E\left[e^{-H(t)\eta}\right] \approx \left[1 + \frac{\sigma^2}{2!} H^2(t)\right] e^{-H(t)\mu}$$

Loglog Distribution – Example

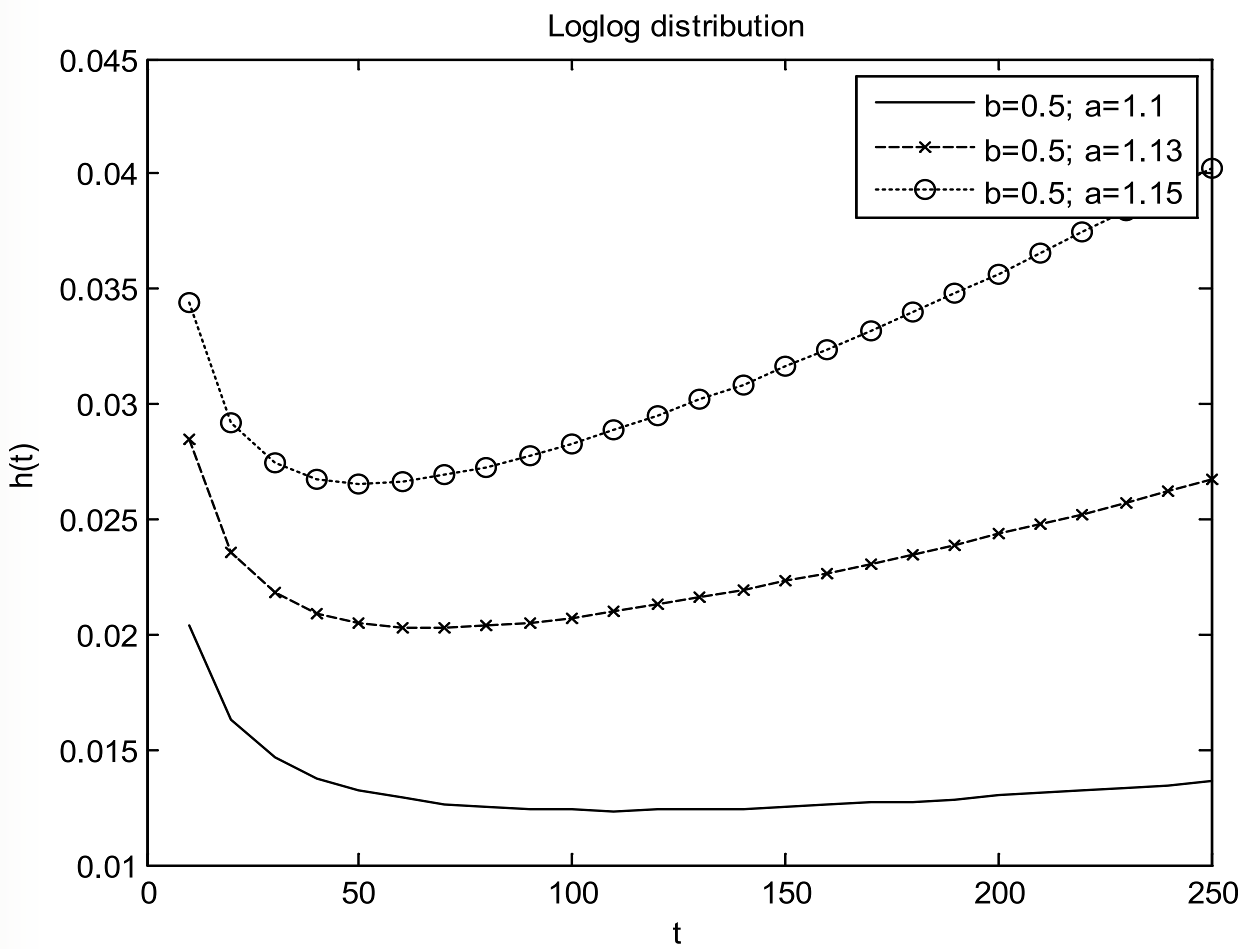
- Assume system lifetime ~ Loglog(a,b) with failure rate

$$h(t) = b \ \ln a \ t^{b-1} \ a^{t^b} \qquad t > 0, \ a > 1, \ b > 0$$

- Assume η ~ gamma(α, β)
- System reliability function

$$R_1(t) = e^{1-a^{t^b}}$$

Failure rate h(t) for various values of a and b = 0.5



Loglog Dist. - Example

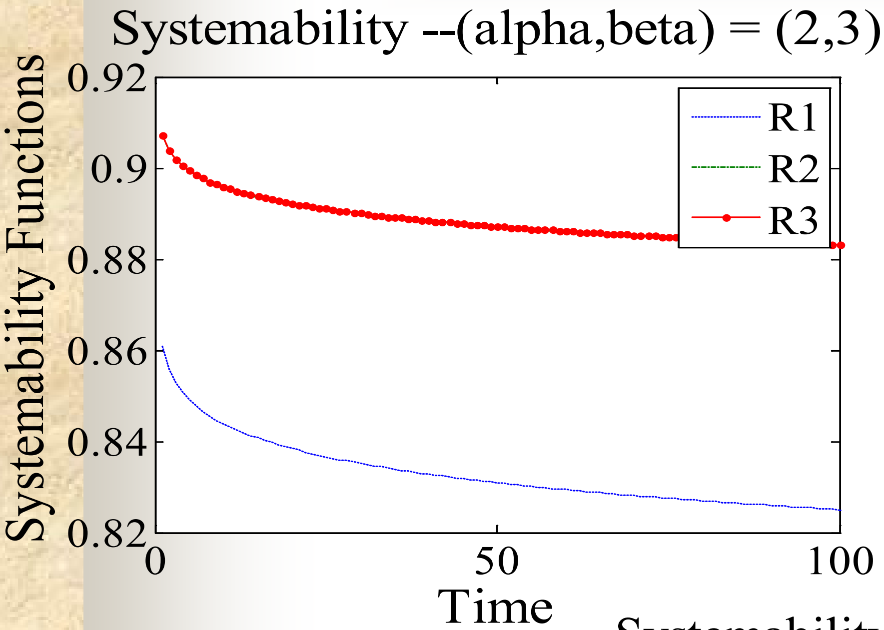
- Systemability function

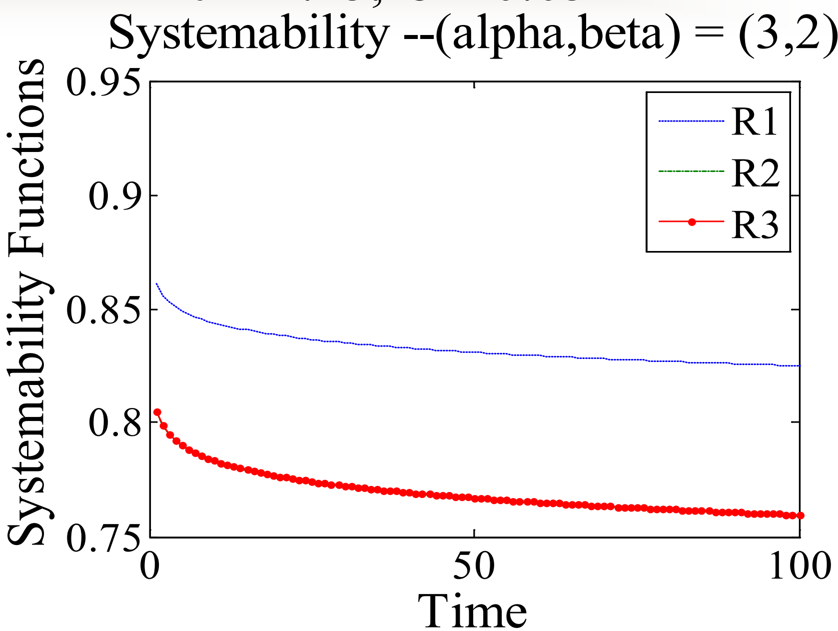
$$R_2(t) = \left(\frac{\beta}{\beta + a^{t^b} - 1} \right)^{\alpha}$$

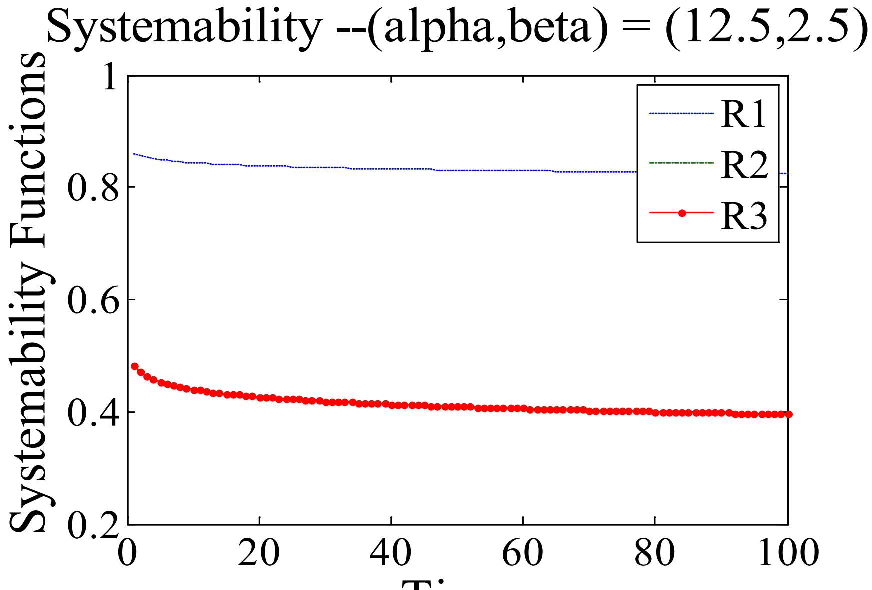
- Systemability approximations

$$R_3(t) = \left(1 + \frac{\alpha \left(a^{t^b} - 1 \right)^2}{2\beta^2} \right) e^{-\frac{\alpha \left(a^{t^b} - 1 \right)}{\beta}}$$

Systemability vs Systemability approximation for $a = 1.15,\ b = 0.05$







What Models Should Be Used?

(H. Pham, “A New Software Reliability Model with Vtub-Shaped Fault-Detection Rate and the Uncertainty of Operating Environments”, **Optimization**, vol 63, 2014: Published online December 2013)

Model Comparison

Model	m(t)
Goel-Okumoto (G-O)	$m(t) = a(1 - e^{-bt})$
Delayed S-shaped	$m(t) = a(1 - (1 + bt)e^{-bt})$
Inflection S-shaped	$m(t) = \dfrac{a(1 - e^{-bt})}{1 + \beta\, e^{-bt}}$
Yamada Imperfect debugging 1	$m(t) = a[1 - e^{-bt}][1 - \dfrac{\alpha}{b}] + \alpha\, a\, t$
PNZ model	$m(t) = \dfrac{a}{1 + \beta\, e^{-bt}}\left([1 - e^{-bt}][1 - \dfrac{\alpha}{b}] + \alpha t\right)$
Pham-Zhang model	$m(t) = \dfrac{1}{1 + \beta e^{-bt}}\left((c + a)(1 - e^{-bt}) - \dfrac{a}{b - \alpha}(e^{-\alpha t} - e^{-bt})\right)$
Dependent-parameter model 1	$m(t) = \alpha(1 + \gamma t)(\gamma t + e^{-\gamma t} - 1)$
Dependent-parameter model 2	$m(t) = m_0\left(\dfrac{\gamma t + 1}{\gamma t_0 + 1}\right)e^{-\gamma(t - t_0)} + \alpha(\gamma t + 1)\left[\gamma t - 1 + (1 - \gamma t_0)e^{-\gamma(t - t_0)}\right]$
Vtub-shaped fault-detection rate model	$m(t) = N\left(1 - \left(\dfrac{\beta}{\beta + a^{t^b} - 1}\right)^{\alpha}\right)$

Criteria For Model Selection

MSE: measures the deviation between the predicted values with the actual observation

$$\text{MSE} = \frac{\sum_{i=1}^{n} (\hat{m}(t_i) - y_i)^2}{n - l}$$

Predictive ratio risk (PRR): measures the distance of model estimates from the actual data against the model estimate

$$PRR = \sum_{i=1}^{n} \left(\frac{\hat{m}(t_i) - y_i}{\hat{m}(t_i)} \right)^2$$

Predictive power (PP): the distance of model estimates from the actual data against the actual data

$$PP = \sum_{i=1}^{n} \left(\frac{\hat{m}(t_i) - y_i}{y_i} \right)^2$$

Criteria For Model Selection

MSE: measures the deviation between the predicted values with the actual observation

$$\text{MSE} = \frac{\sum_{i=1}^{n} (\hat{m}(t_i) - y_i)^2}{n - d}$$

Predictive ratio risk (PRR): measures the distance of model estimates from the actual data against the model estimate

$$PRR = \sum_{i=1}^{n} \left(\frac{\hat{m}(t_i) - y_i}{\hat{m}(t_i)} \right)^2$$

Predictive power (PP): the distance of model estimates from the actual data against the actual data

$$PP = \sum_{i=1}^{n} \left(\frac{\hat{m}(t_i) - y_i}{y_i} \right)^2$$

Normalized Criteria Distance (NCD) value, D_k, measures the distance of the normalized criteria from the origin for kth model where W_j denotes the weight of the criterion j for j = 1,2,…,d

$$D_k = \sqrt{\left(\sum_{j=1}^{d} \left(\left(\frac{C_{kj}}{\sum_{i=1}^{s} C_{ij}} \right)^2 w_j \right) \right)}$$

Software System Test Data (*System Software Reliability*,2006)

Week index	Exposure time (Cum. system test hours)	Fault	Cum. fault
1	416	3	3
2	832	1	4
3	1248	0	4
4	1664	3	7
5	2080	2	9
6	2496	0	9
7	2912	1	10
8	3328	3	13
9	3744	4	17
10	4160	2	19
11	4576	4	23
12	4992	2	25
13	5408	5	30
14	5824	2	32
15	6240	4	36
16	6656	1	37
17	7072	2	39
18	7488	0	39
19	7904	0	39
20	8320	3	42
21	8736	1	43

Model Comparisons & Results

Model / Criteria	MSE (Rank)	PRR (Rank)	PP (Rank)
1. G -O Model	6.61 (7)	0.69 (1)	1.10 (7)
2. Delayed S-shaped	3.27 (5)	44.27 (8)	1.43 (8)
3. Inflection S-shaped	1.87 (2)	5.94 (5)	0.90 (4)
4. Yamada imperfect debugging model	4.98 (6)	4.30 (4)	0.81 (3)
5. PNZ model	1.99 (3)	6.83 (7)	0.96 (6)
6. Pham-Zhang model	2.12 (4)	6.79 (6)	0.95 (5)
7. Dependent-parameter model 1	43.69 (9)	601.34 (9)	4.53 (9)
8. Dependent-parameter model 2	24.79 (8)	1.14 (2)	0.73 (1)
9. Vtub-shaped fault-detection rate model	1.80 (1)	2.06 (3)	0.77 (2)

Model Comparisons & Results (cont.)

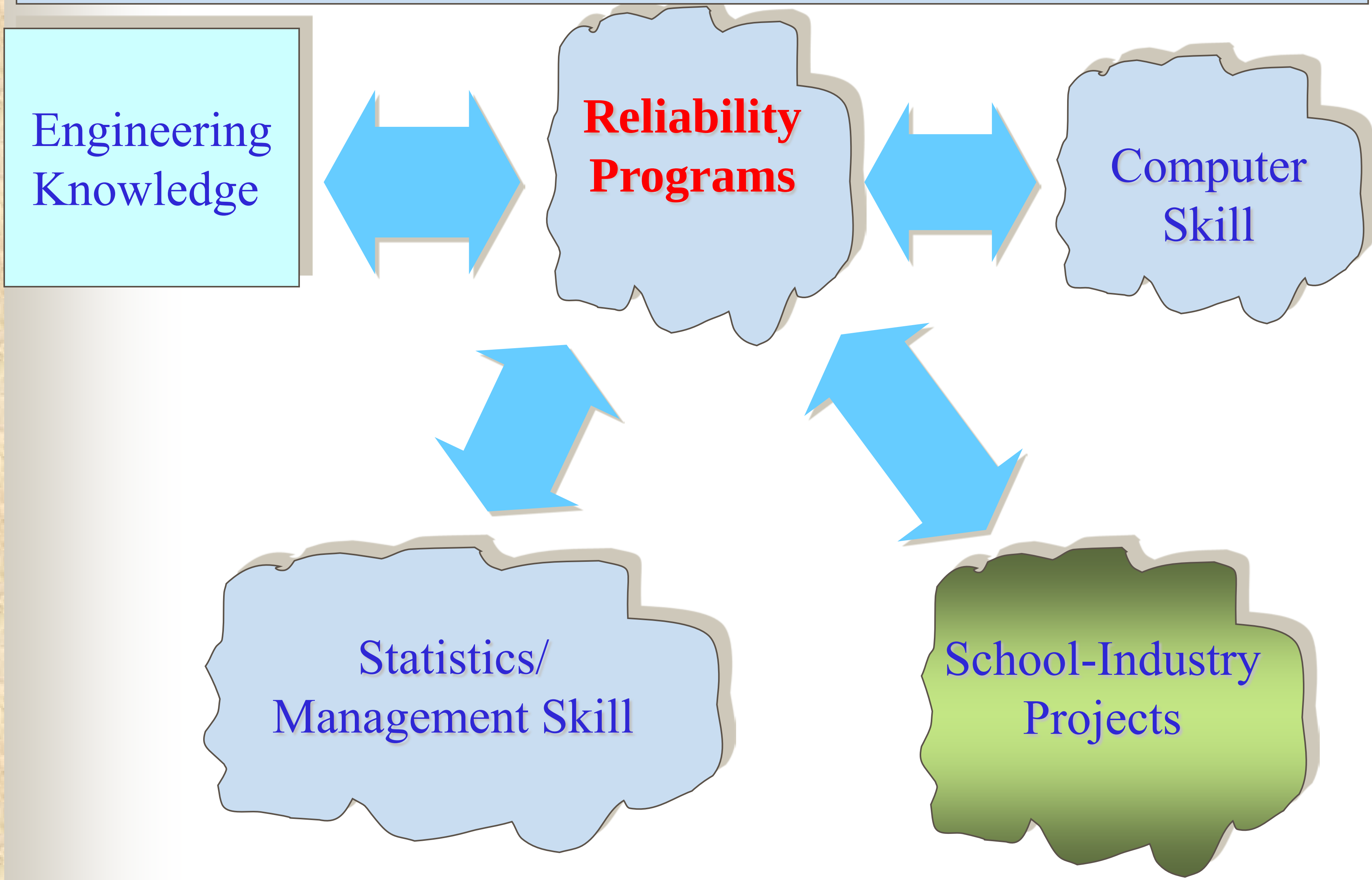
Model / Criteria	MSE (Rank)	PRR (Rank)	PP (Rank)	NCD Value (D_k)	Model Rank
1. G -O Model	6.61 (7)	0.69 (1)	1.10 (7)	0.115843	6
2. Delayed S-shaped	3.27 (5)	44.27 (8)	1.43 (8)	0.139264	7
3. Inflection S-shaped	1.87 (2)	5.94 (5)	0.90 (4)	0.077194	2
4. Yamada imperfect debugging model	4.98 (6)	4.30 (4)	0.81 (3)	0.086315	5
5. PNZ model	1.99 (3)	6.83 (7)	0.96 (6)	0.082414	4
6. Pham-Zhang model	2.12 (4)	6.79 (6)	0.95 (5)	0.082015	3
7. Dependent-parameter model 1	43.69 (9)	601.34 (9)	4.53 (9)	1.079700	9
8. Dependent-parameter model 2	24.79 (8)	1.14 (2)	0.73 (1)	0.278587	8
9. Vtub-shaped fault-detection rate model	1.80 (1)	2.06 (3)	0.77 (2)	0.066303	1

Reliability Opportunities: Big Data!

High Tech Companies in the past 20 years!

- Amazon Inc. Founded: 1994
- Yahoo Founded: 1994
- eBay Founded: 1995
- Google Founded: 1998
- Facebook, Inc. Founded: 2004
- YouTube Founded: 2005
- Twitter Inc. Founded: 2006

Knowledge That Should Be Covered in Reliability Programs



Have a Wonderful Day!